

8 Iloczyn skalarny

1. Obliczyć iloczyny skalarne wektorów

- (a) $\mathbf{v}_1 = (3, 2, 0), \mathbf{v}_2 = (-1, 2, 1)$
- (b) $\mathbf{v}_1 = (-5, 2, 3), \mathbf{v}_2 = (2, 0, 8)$
- (c) $\mathbf{v}_1 = (1, 3, 4), \mathbf{v}_2 = (-3, 2, 0)$
- (d) $\|\mathbf{v}_1\| = 3, \|\mathbf{v}_2\| = 11, \angle(\mathbf{v}_1, \mathbf{v}_2) = \frac{\pi}{6}$
- (e) $\|\mathbf{v}_1\| = 10, \|\mathbf{v}_2\| = 2, \angle(\mathbf{v}_1, \mathbf{v}_2) = \frac{\pi}{4}$
- (f) $\|\mathbf{v}_1\| = 3, \|\mathbf{v}_2\| = 9, \angle(\mathbf{v}_1, \mathbf{v}_2) = \frac{\pi}{2}$

2. Wyznaczyć $(\mathbf{v}_1|\mathbf{v}_2), \|\mathbf{v}_1\|, \|\mathbf{v}_2\|, \|\mathbf{v}_1 + \mathbf{v}_2\|, \|\mathbf{v}_1 - \mathbf{v}_2\|$

- (a) $\mathbf{v}_1 = (5, 3), \mathbf{v}_1 = (0, 3)$
- (b) $\mathbf{v}_1 = (3, 2), \mathbf{v}_1 = (-9, -6)$
- (c) $\mathbf{v}_1 = (2, 1, 0), \mathbf{v}_1 = (1, -2, 3)$
- (d) $\mathbf{v}_1 = (2, -3, 1), \mathbf{v}_1 = (1, 1, 2)$
- (e) $\mathbf{v}_1 = (1, 0, -2, 1), \mathbf{v}_1 = (-1, 2, 1, 3)$
- (f) $\mathbf{v}_1 = (1, 0, 2, 1), \mathbf{v}_1 = (1, 2, 1, 3)$

3. Niech $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ będą wektorami z przestrzeni Euklidesa, takimi że

$(\mathbf{v}_1|\mathbf{v}_2) = 2, (\mathbf{v}_1|\mathbf{v}_3) = -3, (\mathbf{v}_2|\mathbf{v}_3) = 2, \|\mathbf{v}_1\| = 1, \|\mathbf{v}_2\| = 2, \|\mathbf{v}_3\| = 3$. Obliczyć

- (a) $(\mathbf{v}_1 + \mathbf{v}_2|\mathbf{v}_2 + \mathbf{v}_3)$
- (b) $(\mathbf{v}_1 - \mathbf{v}_2 + 3\mathbf{v}_3|2\mathbf{v}_1 + \mathbf{v}_2)$
- (c) $\|\mathbf{v}_1 + \mathbf{v}_2\|$
- (d) $\|\mathbf{v}_1 - 2\mathbf{v}_2 + 4\mathbf{v}_3\|$

4. W przestrzeni Euklidesa dane są wektory $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, takie że

$(\mathbf{v}_1|\mathbf{v}_2) = 10, (\mathbf{v}_1|\mathbf{v}_3) = -2, (\mathbf{v}_2|\mathbf{v}_3) = 3, \|\mathbf{v}_1\| = 2, \|\mathbf{v}_2\| = 4, \|\mathbf{v}_3\| = 5$. Obliczyć

- (a) $(\mathbf{v}_1|\mathbf{v}_2 + \mathbf{v}_3)$
- (b) $(\mathbf{v}_1 + 2\mathbf{v}_2|\mathbf{v}_3)$
- (c) $(\mathbf{v}_3|\mathbf{v}_2 - 10\mathbf{v}_3)$
- (d) $3(\mathbf{v}_1 - 2\mathbf{v}_2|\mathbf{v}_3 + \mathbf{v}_1)$
- (e) $\|\mathbf{v}_1 + 3\mathbf{v}_3\|$
- (f) $\|3\mathbf{v}_3\|$
- (g) $\|\mathbf{v}_3 - 2\mathbf{v}_2\|$

5. Obliczyć kąt pomiędzy wektorami

- (a) $\mathbf{v}_1 = (1, 1, 0, 2), \mathbf{v}_2 = (3, 3, -3, 4)$
- (b) $\mathbf{v}_1 = (3, 2), \mathbf{v}_2 = (0, 1)$
- (c) $\mathbf{v}_1 = (3, 2, 0, 6, 10), \mathbf{v}_2 = (2, 1, 0, 1, 1)$
- (d) $\mathbf{v}_1 = (-3, -2, 1), \mathbf{v}_2 = (-1, 7, 8)$

6. Obliczyć

(a) $(3\mathbf{v}_1 - 5\mathbf{v}_2)(\mathbf{v}_1 + 3\mathbf{v}_2)$, gdy $\|\mathbf{v}_1\| = 1, \|\mathbf{v}_2\| = 2, \angle(\mathbf{v}_1, \mathbf{v}_2) = \frac{\pi}{3}$

(b) $(\mathbf{v}_1 + \mathbf{v}_2)^2(\mathbf{v}_1 - \mathbf{v}_2)^2$, gdy $\|\mathbf{v}_1\| = 5, \|\mathbf{v}_2\| = 5$

(c) $\|3\mathbf{v}_1 - 2\mathbf{v}_2\|$, gdy $\|\mathbf{v}_1\| = 2, \|\mathbf{v}_2\| = 3, \angle(\mathbf{v}_1, \mathbf{v}_2) = \frac{\pi}{3}$