CORRIGENDUM

Capturing an Intruder in a Building

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There is an error in Procedure `Locate` and Lemma 2. The error is that in Procedure `Locate`, Z may become visible to A when A is not on a vertex, and Z's move thereafter cannot be assumed to be random. We noticed this error while discussing a similar algorithm with Adrian Dumitrescu. A revised procedure and Lemma 2 are presented below. The rest of the results in the paper are not affected.

Detecting an Intruder

Define the *backbone* of the grid $G_{n\times n\times n}$ with the *root* $(0,0,0)$ as the set of all points of shaft $(0,0)$, north-south corridor $(0,0)$, and west-east corridor $(0,0)$. Next, define the *xy-plane* to be the set of points (x,y,z) in $G_{n\times n\times n}$ with $z=0$; similarly, define the *yz-plane* and *xz-plane* to be the set of points (x,y,z) with $x=0$ and $y=0$, respectively. (See Figure **??** for an illustration).

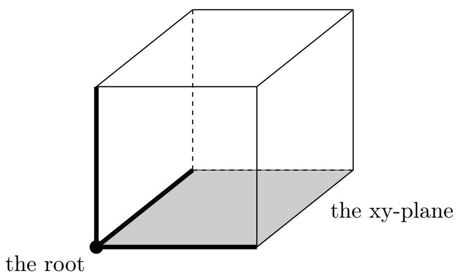

Figure 1: The backbone and the xy-plane of the grid.

A can see Z from a vertex using the revised procedure given below. The idea is to give A a choice of not moving from a vertex.

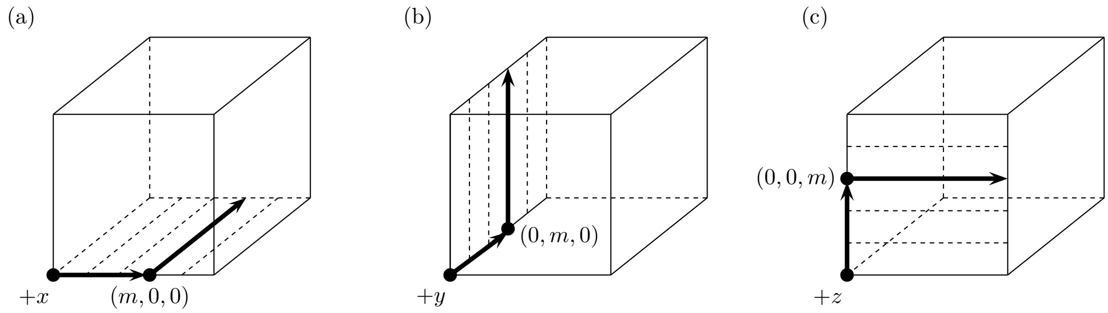

Figure 2: An illustration of procedure `Locate`. (a) A chooses direction $+x$ and crossing $c = (m,0,0)$, moves to $(m,0,0)$ and waits there till time $t = n$; next, he waits there for additional time $w \in [0,2n]$, and at time $t = n + w$, starts moving towards $(m, n-1, 0)$ along north-south corridor $(m,0)$. (b) A chooses direction $+y$ and crossing $c = (0,m,0)$, moves to $(0,m,0)$ and waits there till time $t = n$; next, he waits there for additional time $w \in [0,2n]$, and at time $t = n + w$, starts moving towards $(0, m, n-1)$ along shaft $(0,m)$. (c) A chooses direction $+z$ and crossing $c = (0,0,m)$, moves to $(0,0,m)$ and waits there till time $t = n$; next, he waits there for additional time $w \in [0,2n]$, and at time $t = n+w$, starts moving towards $(n-1, 0, m)$ along east-west corridor $(0,m)$.

Procedure `Locate`

/* We assume that A is at the root $(0,0,0)$ of the backbone, otherwise, A goes to $(0,0,0)$ from its current location at maximum speed s. */

Step 1. Reset time $t = 0$. Uniformly at random, A chooses a direction $d \in \{+x, +y, +z\}$, and next, also uniformly at random, he chooses an integer $m \in \{0, 1, \ldots, n-1\}$. Let c be vertex $(m,0,0)$ if $d = +x$, $(0,m,0)$ if $d = +y$, and $(0,0,m)$ if $d = +z$. (See Figure **??**.) A leaves $(0,0,0)$ at $t = 0$ and goes to c at speed s, and waits at c till $t = n$. (A arrives at c at time $\frac{m}{s}$. Note that if $m = 0$, then A stays at $(0,0,0)$ till $t = n$.)

Step 2. A executes one of the following actions with the given probabilities.

Action 2.1 With probability $\frac{1}{3}$, A remains stationary at c until $t = 4n$.

Action 2.2 With probability $\frac{2}{3}$, A choose a waiting time $w \in [0, 2n]$ uniformly at random, and waits at c for time w. Then, at time $t = n + w$ (see Figure 2(a-c) for an illustration):

(a) Case $d = +x$: A leaves $c = (m,0,0)$, moves along north-south corridor $(m,0)$ to the other end $(m, n-1, 0)$ at speed s, and stays at $(m, n-1, 0)$ till $t = 4n$.

(b) Case $d = +y$: A leaves $c = (0,m,0)$, moves along shaft $(0,m)$ to the other end $(0, m, n-1)$ at speed s, and stays at $(0, m, n-1)$ till $t = 4n$.

(c) Case $d = +z$: A leaves $c = (0,0,m)$, moves along east-west corridor $(0,m)$ to the end $(n-1, 0, m)$ at speed s, and stays at $(n-1, 0, m)$ till $t = 4n$.

Termination Condition `Locate` ends in success at the moment A sees Z from a vertex. If A sees Z from a non-vertex position or A never sees Z in $[0, 4n]$, then `Locate` ends in failure. A executes `Locate` repeatedly until it ends in success.

In the following, we show that the probability of success of one execution of `Locate`, i.e., A sees Z from a vertex in $[0, 4n]$, is at least $\frac{1}{9n}$.

We need the following notation. For a time interval $T = [t_1, t_2]$, $t_1 \le t_2$, we denote by $|T| = t_2 - t_1$ the length of T. For any $t_3 \ge 0$, $T - t_3$ denotes the interval $[t_1 - t_3, t_2 - t_3]$ obtained by shifting T early by t_3.

Case 1. For some $m \in \{0, 1, \ldots, n-1\}$, Z is in one of north-south corridor $(m, 0)$, shaft $(m, 0)$ and west-east corridor $(0, m)$ at some time in $[\frac{m}{s}, n]$, and let t' be the earliest such time. (In this case, at or before time n, Z may gain some knowledge about A's choice.)

Suppose Z is in north-south corridor $(m, 0)$ at t'. With probability at least $\frac{1}{3n}$, A has chosen $c = (m, 0, 0)$ and stays at c in the entire interval $[\frac{m}{s}, n]$. Thus A sees Z from c at time t' with probability at least $\frac{1}{3n}$. The argument for the other two cases are similar.

Case 2. For any $m \in \{0, 1, \ldots, n-1\}$, Z is not in any of north-south corridor $(m, 0)$, shaft $(m, 0)$ and west-east corridor $(0, m)$, at any time in $[\frac{m}{s}, n]$. (In this case, at time n, Z does not have any knowledge about A's choice.)

(a) Suppose Z enters one of the xy-plane, yz-plane, and xz-plane in $[n, 4n]$, and let t' be the earliest such time. Let c be a vertex in the backbone from which Z is visible at t'. Since A stays at c in $[n, 4n]$ with probability at least $\frac{1}{3n} \cdot \frac{1}{3} = \frac{1}{9n}$, A sees Z from c at t' with the same probability.

(b) Suppose Z does not enter any of the xy-plane, yz-plane, and xz-plane in $[n, 4n]$. Consider now two time intervals $W = [n, 3n]$ and $I = [2n, 3n]$. Fix a move of Z in $[n, 4n]$, and for $1 \le i, j, k \le n-1$, let $x_{i,j}$, $y_{i,j}$ and $z_{i,j}$, respectively, be the total time in I during which Z is in shaft (i, j), east-west corridor (j, k) and north-south corridor (i, k).[1] Obviously, we have

$$\sum_{\substack{1 \le i \le n-1 \\ 1 \le j \le n-1}} x_{i,j} + \sum_{\substack{1 \le j \le n-1 \\ 1 \le k \le n-1}} y_{j,k} + \sum_{\substack{1 \le i \le n-1 \\ 1 \le k \le n-1}} z_{i,k} \ge |I|. \tag{1}$$

(When Z is at a vertex, he is simultaneously in the shaft and two corridors passing through it.) Now, for some fixed i and j, let $J_1, J_2, \ldots, J_l$ be the disjoint maximal intervals in I in which Z is in shaft (i, j). Then $|J_1| + |J_2| + \cdots + |J_l| = x_{i,j}$. Suppose in Step 1, A selects direction $+x$ (with probability $\frac{1}{3}$) and $m = i$ (with probability $\frac{1}{n}$), and executes Action 2.2 (with probability $\frac{2}{3}$). Since it takes exactly $\frac{j}{s}$ time units for A to go from $c = (i, 0, 0)$ to $(i, j, 0)$, A will see Z in shaft (i, j) at the moment he reaches $(i, j, 0)$ if A leaves vertex $c = (i, 0, 0)$ at any time in any of $J_1 - \frac{j}{s}$, $J_2 - \frac{j}{s}$, $\ldots$, $J_l - \frac{j}{s}$. Since $1 \le t - \frac{j}{s} < 3n$ holds for any $2n \le t \le 3n$ and $1 \le j \le n-1$, these intervals are pairwise disjoint sub-intervals of $W = [n, 3n]$. Then, since A's starting time $n + w$ is chosen uniformly at random in W (because w is chosen uniformly at random in $[0, 2n]$), the probability of the above event is

$$\frac{\sum_{1 \le t \le l} |J_t - \frac{j}{s}|}{|W|} = \frac{\sum_{1 \le t \le l} |J_t|}{|W|} = \frac{x_{i,j}}{|W|}.$$

Consequently, assuming that A has chosen $d = +x$ and $m = i$ in Step 1 and Action 2.2 in Step 2, the possibility that A sees Z from a vertex while moving along north-south corridor $(i, 0)$ is at least

$$\sum_{1 \le j \le n-1} \frac{x_{i,j}}{|W|}.$$

[1] We take $i, j, k \ge 1$ since by assumption Z never enters any of the xy-plane, yz-plane, and xz-plane in $[n, 4n]$.

(If $s = 1$, then A may see Z in multiple shafts while moving along north-south corridor $(i, 0)$. This means that two sub-intervals of W contributing in the above summation, associated with different values of j, may not be disjoint, sharing a single point in time. However, for any given move of Z, the set of starting times for A that make this possible has measure zero, and hence the above claim always holds.) Using a similar argument for axis directions $+y$ and $+z$ and summing all up, the probability that A chooses Action 2.2 and sees Z from a vertex is, by (1), at least

$$\frac{1}{3n} \cdot \frac{2}{3} \cdot \Big(\sum_{\substack{1 \le i \le n-1 \\ 1 \le j \le n-1}} \frac{x_{i,j}}{|W|} + \sum_{\substack{1 \le j \le n-1 \\ 1 \le k \le n-1}} \frac{y_{j,k}}{|W|} + \sum_{\substack{1 \le i \le n-1 \\ 1 \le k \le n-1}} \frac{z_{i,k}}{|W|} \Big) \ge \frac{1}{3n} \cdot \frac{2}{3} \cdot \frac{|I|}{|W|} = \frac{1}{9n}.$$

Consequently, we obtain the following lemma.

Lemma 2 *In $G_{n \times n \times n}$, with probability at least $\frac{1}{9n}$, a single robot with a maximum speed of $s \ge 1$ can detect/see Z from a vertex within $O(n)$ time.*